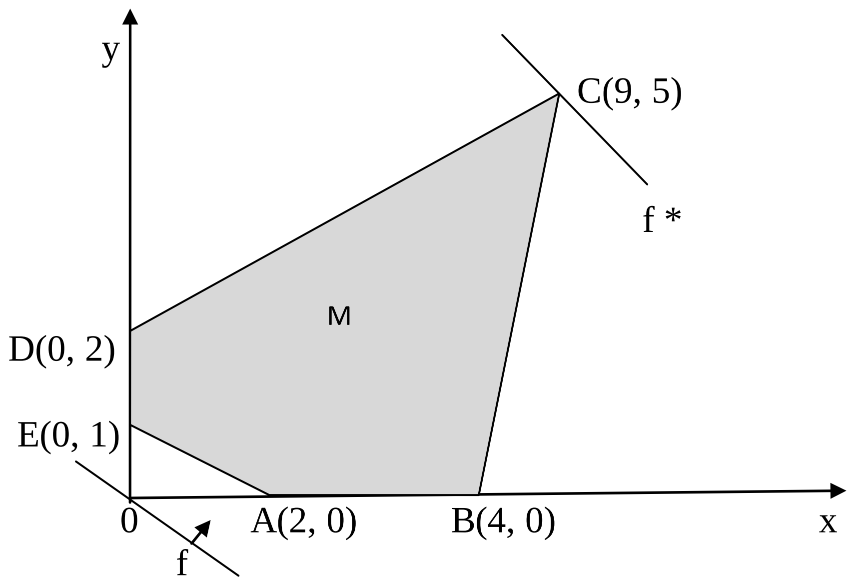
Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Qualifying Studies (Mathematics)		Final Term Exam Date: 1 / 4 / 2013 Operations Research Duration: 1 hours

(1)Solve the following LP problems graphically: maximize $f = x + 2y$ s.t $x + 2y \geq 2,\quad -x + 3y \leq 6,\quad x - y \leq 4,\quad x, y \geq 0$	15
(2) minimize $f = x - y - z$ s.t $2x - y + z \leq 4,\quad x + 2y + 2z \leq 10,\quad -x + y - z \leq 8,\quad x, y, z \geq 0$	15
(3) maximize $f = x + y + z - u$ s.t $x - y + z - u \leq 4,\quad x + y - z + u \geq 6,\quad x, y, z, u \geq 0$	20

Good Luck *Dr. Mohamed Eid*

Answer

(1)



The line $x + 2y = 2$, when $x = 0$ then $y = 1$ and when $x = 2$ then $y = 0$. Then it passes through the points (0, 1), (2, 0).
The line $-x + 3y = 6$, when $x = 0$ then $y = 2$ and when $x = 3$ then $y = 3$. Then it passes through the points (0, 2), (3, 3).
The line $x - y = 4$, when $x = 0$ then $y = -4$ and when $x = 4$ then $y = 0$.
Then it passes through the points (0, – 4), (4, 0).

Then, we determine the feasible domain M of vertices: A(2,0), B(4,0), C(9,5), D(0,2) and E(0,1), see the figure.
The equation of the objective function x + 2y = 0, when x = 2 then y = –1. Then it passes through the points (0, 0) and (2, –1) and can be traced as in figure.
Since the coefficients of the objective function f are 1 and 2. Then the point(1, 2) lies in the first quarter which is the increasing direction of f. Then, the last point of intersection of the feasible domain M and the objective function f is the vertex C(9, 5) which is the optimal solution. The optimal value of f is 19.

(2)The standard form of this problem is:

minimize $f = x - y - z$

s.t $2x - y + z + s_1 = 4$

$x + 2y + 2z + s_2 = 10$

$-x + y - z + s_3 = 8$, $x, y, z, s_1, s_2, s_3 \geq 0$

The steps of the simplex method goes as follows:

B.V	x	y	z	s_1	s_2	s_3	Solu
s_1	2	–1	1	1	0	0	4
s_2	1	2	2	0	1	0	10
s_3	–1	1	–1	0	0	1	8
f	–1	1	1	0	0	0	0
z	2	–1	1	1	0	0	4
s_2	–3	4	0	–2	1	0	2
s_3	1	0	0	1	0	1	12
f	–3	2	0	–1	0	0	- 4
z	5/4	0	1	1/2	1/4	0	9/2
y	–3/4	1	0	–1/2	1/4	0	1/2
s_3	1	0	0	1	0	1	12
f	–3/2	0	0	0	–1/2	0	–5

This is the optimum case. Then the optimal solution is:

$(x^*, y^*, z^*) = (0, 1/2, 9/2)$ or $(0, 5, 0)$ with optimal value $f^* = -5$

(3)The standard form of this problem is:

maximize $f = x + y + z - u$

s.t $x - y + z - u + s_1 = 4$

$$x + y - z + u - t + v = 6, \quad x, y, z, u, s_1, t, v \geq 0$$

where s_1 are slack variable, t is surplus variable and v is artificial variable.

Let w = v. Then, the objective of phase one is:

$$w + x + y - z + u - t = 6$$

The steps of phase one goes as table:

B.V	x	y	z	u	t	s_1	v	Solu
s_1	1	-1	1	-1	0	1	0	4
v	1	1	-1	1	-1	0	1	6
f	-1	-1	-1	1	0	0	0	0
w	1	1	-1	1	-1	0	0	6
x	1	-1	1	-1	0	1	0	4
v	0	2	-2	2	-1	-1	1	2
f	0	-2	0	0	0	1	0	4
w	0	2	-2	2	-1	-1	0	2
x	1	0	0	0	$-1/2$	$1/2$	$1/2$	5
y	0	1	-1	1	$-1/2$	$-1/2$	$1/2$	1
f	0	0	-2	2	-1	0	1	6
w	0	0	0	0	0	0	-1	0

This is the end of phase one. Phase two starts with the following table which is formed by deleting the column of v and the w-row.

B.V	x	y	z	u	t	s_1	Solu
x	1	0	0	0	$-1/2$	$1/2$	5
y	0	1	-1	1	$-1/2$	$1/2$	1
f	0	0	-2	2	-1	0	6

There is no optimal solution because the coefficient of z in f-equation is negative but the pivoting operation can not be cared.

The feasible solution is $(x, y, z, u) = (5,1, 0, 0)$ with value $f^* = 6$.